



# **Fundamentals of the Dempster-Shafer Theory and its Applications to System Safety and Reliability Modelling**

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# Dempster-Shafer Applications

## Introduction

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### Objective

- Modelling & expressing **uncertainties** in safety & reliability analyses
- Evidence measures offer a **different kind of flavour** to RAMS engineers

### What's new?

- Evidence measures **belief** and **plausibility** are applied instead of → probabilities  
instead of → membership function (fuzzy set theory)

### What's not new?

- Methods introduced (FTA, ETA, RCM, FMECA)
- Fundamentals of the **Dempster-Shafer Theory**

# Dempster-Shafer Applications

## Introduction

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### Note

- ESREL 2007 → special DS approach tailored to RCM
- SSARS 2007 → general DS approach to Safety & Reliability Modelling  
→ more details

### Disclaimer

- Nobody is forced to apply evidence measures
- Not faster, bigger, better, higher → just different

# Outline

## Introduction

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### Part 1 – Fundamentals

- History
- Scenario
- Interpretations

### Part 2 – Illustration

- The DS calculus in eight steps

### Part 3 – Applications to System Safety & Reliability Modelling

- FTA – Fault Tree Analysis
- ETA – Event Tree Analysis
- RCM – Reliability-centred Maintenance
- Further Analyses

### Part 4 – Outroduction

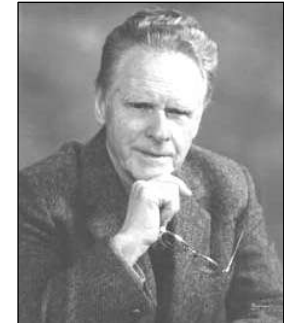
- Pros & Cons

# History

## Introduction

### 📖 1966 – Arthur P. Dempster

- Developed Theory
- “Upper & lower **probabilities**”
- Suitable to express uncertain expert judgements



A. P. Dempster

### 📖 1976 – Glenn Shafer

- Extended, refined, recast
- “Upper **probabilities** & degrees of **belief**”
- “DS Theory of Evidence”, “DS Evidential Theory” → DST



G. Shafer

### 📖 1988 – George J. Klir & Tina A. Folger

- Introduce → “Degrees of **belief** & **plausibility**”
- Evidence measures depart from being probabilities

# The Scenario

## Fundamentals

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### Scenario

- System
- Hypotheses
- Frame of discernment
- Pieces of evidence
- Data sources

### System

- Borders
- In- and outputs
- Elements (e.g. components or modules)
- **Links** between the elements
- **Interactions** of the elements
- Task of the system

# The Scenario

## Fundamentals

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### Scenario

- System
- Hypotheses
- Frame of discernment
- Pieces of evidence
- Data sources

### Hypotheses

- Single hypothesis → e.g. represents one **state**, one **answer**
- Example → “functioning”, “marginal”, “faulty”
- Example → “yes”, “uncertain”, “no”

### Properties

- Unique **and**
- not overlapping **and**
- mutually exclusive

# The Scenario

## Fundamentals

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### Scenario

- System
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- Data sources

### Frame of discernment

- Representation → universal set  $\Omega$
- Hypotheses → elements of frame of discernment
- $\Omega = \{\text{"functioning"}, \text{"marginal"}, \text{"faulty"}\}$
- Power set  $2^\Omega$  → set of all subsets
- Power set  $2^\Omega$  → single **and conjunctions** of hypotheses



# The Scenario

## Fundamentals

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### Scenario

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### Pieces of Evidence

- Symptoms or **events** → e.g. failures
- Assignment  
{evidence → hypothesis(es)} **corresponds to** {cause → consequence(s)}
- Assignment  
{1 p-of-e} **assigned to** {1 hypothesis} or {1 set of hypotheses}  
{>1 p-of-e} **may not be assigned\*** to {same hypothesis}, {same set}

\*) by the same data source

# The Scenario

## Fundamentals

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### Scenario

- System
- Hypotheses
- Frame of discernment
- Pieces of evidence
- Data sources

### Data Sources

- Information provider → experts, empirical studies, data
- Task → quantifying strength {p-of-e → hypothesis} assignments →  $m(A)$
- Requirements
  - free from bias (esp. experts)
  - representative (esp. studies)
  - no source is more important than another one

# The Scenario

## Fundamentals

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### Scenario

- System
- Hypotheses
- Frame of discernment
- Pieces of evidence
- Data sources

### Data Sources

- Information provider → experts
- Expert group
  - Safety → system eng., software eng., reliability eng., service eng.
  - RCM → service eng., maintenance personnel, reliability eng.
- Task → give subjective quantifiable statements
- Basis → data, experience, intuition ... ← biased?

# The Scenario

## Fundamentals

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### Scenario

- System
- Hypotheses
- Frame of discernment
- Pieces of evidence
- Data sources

### Pieces of Evidence

- Expert group → piece of evidence?
  - expert judgement (experience, intuition)
  - experts' subjectivity
- Critical issue?

# Objectivity versus Subjectivity

## Fundamentals

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### Objectivity

- **Exactly** one single hypothesis is **objectively** true

### Subjectivity

- **Uncertain** which hypothesis fits **subjectively** best to reality

### Dempster-Shafer Theory

- Calculus describes & quantifies the **subjective** viewpoint as an assessment for an unknown **objective** fact

### Safety & Reliability Engineering

- PSAM/ESREL 2004 → hypotheses
  - “component  $i$  is functioning” **and** “component  $i$  is faulty”
  - ... same set?!

# Assignments & Sets

## Fundamentals

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### Sets

- $\Omega$  universal set
- $A, B, Z \subseteq \Omega$  sets, containing a **single** hypothesis or a **set** of hypotheses

### Basic Assignment

- $m, m(A)$  quantifies if the element belongs **exactly** to the set  $A$
- $m: 2^\Omega \rightarrow [0, 1]$  mapping (prob.  $\Omega \rightarrow [0, 1]$ )
- $\sum_{A \subseteq \Omega} m(A) = 1$  all statements of an expert are **normalised**
- $m(A) > 0$  focal element, only substantial statements
- $m(\emptyset) = 0$  simplicity (**not** required)

### Differences in Properties to Probabilities

- $m(\Omega) = 1$  **not** required
- $m(A)$  vs.  $m(\neg A)$  **no** relationship required
- If  $A \subset B \subseteq \Omega$ , then  $m(A) \leq m(B)$  **not** required

# The Basic Assignment

## Fundamentals

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### Interpretation of $m$

- Task of  $m \rightarrow$  assign evidential weight to hypothesis(es)  $\rightarrow A \subseteq \Omega$
- Mathematical interpretation of  $m \rightarrow$  “evidential weight”
- Probability  $\rightarrow$  **no** concept, **no** interpretation (📖 ESREL'05 Proceedings)

### Denotations of $m$

- “Basic probability assignment”  $\leftarrow$  no probability
- “Basic belief assignment”  $\leftarrow$  conflicts *belief* measure
- “Basic structure”  $\leftarrow$  conflicts Boolean *structure* function
- “Mass assignment function”  $\leftarrow$  mass confuses in engg. applications
- “Basic assignment”  $\leftarrow$  😊

# Evidential Functions

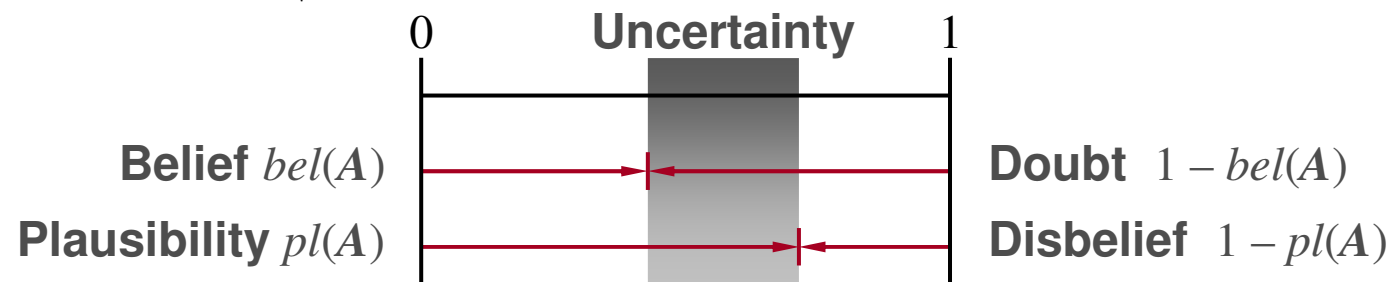
## Fundamentals

### Belief Measure $bel(A)$

- **Belief** is the degree of evidence that the element in question belongs to the set  $A$  as well as to the various special **subsets** of  $A$ .
- $bel(A) = \sum_{B \subseteq A; B \neq \emptyset} m(B)$

### Plausibility Measure $pl(A)$

- **Plausibility** is the degree of evidence that the element in question belongs to the set  $A$  or to any of its **subsets** or to any **set that overlaps** with  $A$ .
- $pl(A) = \sum_{B \cap A \neq \emptyset} m(B)$





# Complements

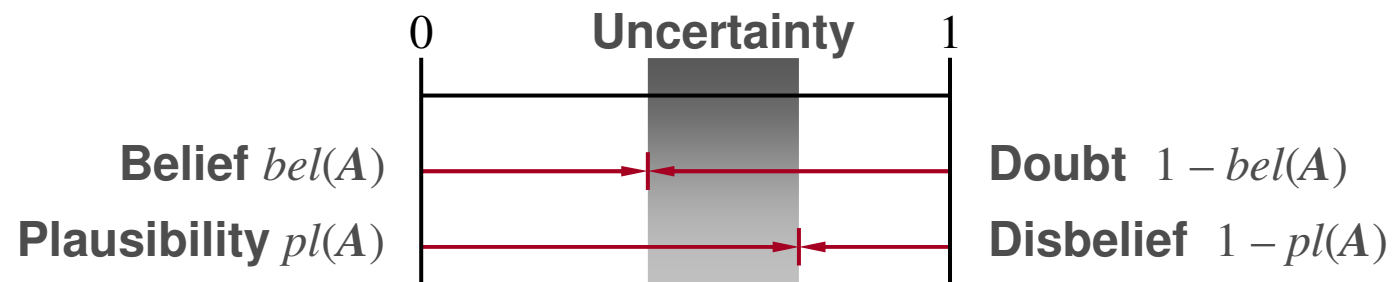
## Fundamentals

### Direct Complements

- Belief versus doubt
- Plausibility versus disbelief

### Contextual Complements

- Certainty → belief versus disbelief
- Uncertainty included → plausibility versus doubt



# Phenomena

## Fundamentals

### Difference in Concepts – Existence of Phenomena

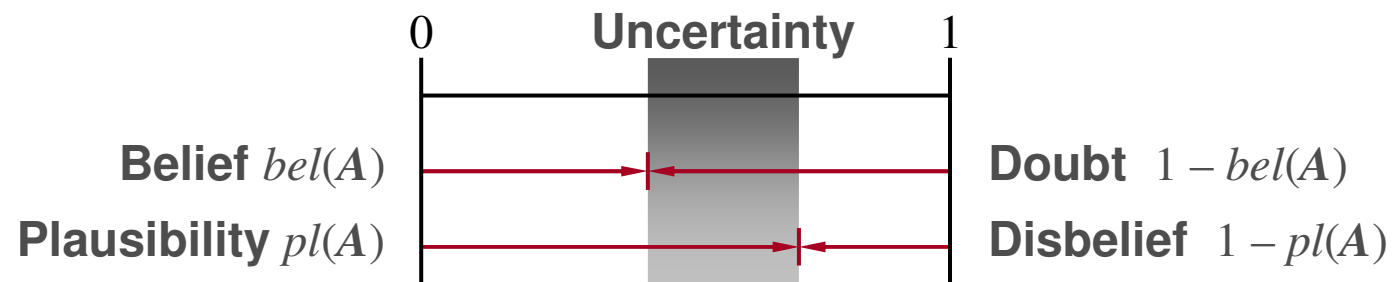
- Evidential measures  
No causal relationship between belief in **existence** and belief in **non-existence**
- Probabilities  
The belief in **existence** implies belief in **non-existence**

$$bel(A)$$

$$bel(\neg A) = 1 - pl(A)$$

$$pr(x_i = 1)$$

$$pr(x_i = 0) = 1 - pr(x_i = 1)$$



# Outline

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# Context

## Illustration

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### Typical Situation in a Power Plant

- Operators @control panel → detect serious changes of system properties
- Causes → failures detectable
- Consequence → system fault  
→ neither be **determined exactly** nor **interpreted certainly**
- Widely discussed (📖 ATHEANA Report, 📖 Eric Hollnagel, etc.)

### DST Approach

- Collects pieces of evidence
- Postulates hypotheses
- Proposes conclusions
- Dempster-Shafer approach → supports operators in reasoning
- Objective → to avoid an **error forcing context**

# Procedure

## Illustration

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### Eight Steps

- Step ① – Creating the Scenario
- Step ② – Quantification of Statements
- Step ③ – Combining Hypotheses
- Step ④ – Reducing the Combination Table
- Step ⑤ – Calculating Products & Sums of Combined Basic Assignments
- Step ⑥ – Combining Basic Assignments
- Step ⑦ – Evidence Measures of Combined Hypotheses
- Step ⑧ – Interpretation

# Creating the Scenario

## Illustration – Step ①

### Scenario

- System → power plant
- Data sources → operators → 2 persons
- Pieces of evidence → failures detected → 4 considered
- Hypotheses → system fault states → 3 considered
- Frame of discernment →  $\Omega = \{h_1, h_2, h_3\}$

### Qualitative Failure-fault(s) Assignments

- 1<sup>st</sup> operator →  $h_1, h_2$  consequences
- 2<sup>nd</sup> operator →  $h_1, h_3$  consequences
- **Same** p-o-e, **different** hypotheses

### DST Restrictions

- No more than one failure lead to the same fault (hypothesis)\*

\*) each data source

Op.	Failure	Fault(s)
1 <sup>st</sup>	$ev_1$	$h_1$
	$ev_2$	$h_2$
	$ev_3$	$h_1, h_2$
	$ev_4$	$h_1, h_2, h_3$
2 <sup>nd</sup>	$ev_1$	$h_1$
	$ev_2$	$h_3$
	$ev_3$	$h_1, h_3$
	$ev_4$	$h_1, h_2, h_3$

# Quantification of Statements

## Illustration – Step ②

### Quantification

- Operators quantify statements, basis  $\rightarrow$  data, intuition & experience
- $m(A_k) = 0 \rightarrow$  no focal element

### Belief

- Example: 1<sup>st</sup> Operator, set  $\{h_1 \cup h_2\}$
- Set and all its subsets
- $\{h_1\}, \{h_2\}, \{h_1 \cup h_2\} \subseteq \{h_1 \cup h_2\}$
- $bel(A_4) = m(A_1) + m(A_2) + m(A_4) = 0.9$

### Plausibility

- At least **1** hypothesis in common
- $\{h_1\}, \{h_2\}, \{h_1 \cup h_2\}, \{h_1 \cup h_3\},$   
 $\{h_2 \cup h_3\}, \{h_1 \cup h_2 \cup h_3\}$   
 $\cap \{h_1 \cup h_2\} \neq \emptyset$
- $pl(A_4) = m(A_1) + m(A_2) + m(A_4)$   
 $+ m(A_5) + m(A_6) + m(A_7) = 1$

1 <sup>st</sup> operator	$2^\Omega$	2 <sup>nd</sup> operator
$m(A_1) = 0.2$	$\{h_1\}$	$m(B_1) = 0.2$
$m(A_2) = 0.1$	$\{h_2\}$	$m(B_2) = 0$
$m(A_3) = 0$	$\{h_3\}$	$m(B_3) = 0.2$
$m(A_4) = 0.6$	$\{h_1 \cup h_2\}$	$m(B_4) = 0$
$m(A_5) = 0$	$\{h_1 \cup h_3\}$	$m(B_5) = 0.4$
$m(A_6) = 0$	$\{h_2 \cup h_3\}$	$m(B_6) = 0$
$m(A_7) = 0.1$	$\{h_1 \cup h_2 \cup h_3\}$	$m(B_7) = 0.2$

# Quantification of Statements

## Illustration – Step ②

### Results

- Input by operators  $\rightarrow m(A_k)$
- Output by calculus  $\rightarrow bel(A_k), pl(A_k)$

$m(A_k)$	$bel(A_k)$	$pl(A_k)$	$2^\Omega$	$m(B_k)$	$bel(B_k)$	$pl(B_k)$
<b>0.2</b>	0.2	0.9	$\{h_1\}$	<b>0.2</b>	0.2	0.8
<b>0.1</b>	0.1	0.8	$\{h_2\}$	<b>0</b>	0	0.2
<b>0</b>	0	0.1	$\{h_3\}$	<b>0.2</b>	0.2	0.8
<b>0.6</b>	0.9	1	$\{h_1 \cup h_2\}$	<b>0</b>	0.2	0.8
<b>0</b>	0.2	0.9	$\{h_1 \cup h_3\}$	<b>0.4</b>	0.8	1
<b>0</b>	0.1	0.8	$\{h_2 \cup h_3\}$	<b>0</b>	0.2	0.8
<b>0.1</b>	1	1	$\Omega$	<b>0.2</b>	1	1



# Combining Hypotheses

## Illustration – Step ③

### Combination

- Combining each set of hypotheses of both operators
- Building cut sets  $\cap$  of both

$\cap$	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$
$B_1$	$h_1$	$\emptyset$	$\emptyset$	$h_1$	$h_1$	$\emptyset$	$h_1$
$B_2$	$\emptyset$	$h_2$	$\emptyset$	$h_2$	$\emptyset$	$h_2$	$h_2$
$B_3$	$\emptyset$	$\emptyset$	$h_3$	$\emptyset$	$h_3$	$h_3$	$h_3$
$B_4$	$h_1$	$h_2$	$\emptyset$	$h_1 \cup h_2$	$h_1$	$h_2$	$h_1 \cup h_2$
$B_5$	$h_1$	$\emptyset$	$h_3$	$h_1$	$h_1 \cup h_3$	$h_3$	$h_1 \cup h_3$
$B_6$	$\emptyset$	$h_2$	$h_3$	$h_2$	$h_3$	$h_2 \cup h_3$	$h_2 \cup h_3$
$B_7$	$h_1$	$h_2$	$h_3$	$h_1 \cup h_2$	$h_1 \cup h_3$	$h_2 \cup h_3$	$\Omega$

# Reducing the Combination Table

## Illustration – Step ④

### Combination

- Objective → avoid mathematical effort
- Drop rows & columns → non-focal elements

$$m(A_k) = 0, m(B_k) = 0$$

$\cap$	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$	$A_6$	$A_7$
$B_1$	$h_1$	$\emptyset$	$A_1$	$A_2$	$A_4$	$A_7$	$h_1$
$B_2$	$\emptyset$	$h_2$	$\emptyset$	$h_2$	$\emptyset$	$h_2$	$h_2$
$B_3$	$\emptyset$	$B_1$	$h_3$	$\emptyset$	$h_1 h_3$	$h_1 h_3$	$h_3$
$B_4$	$h_1$	$B_3$	$\emptyset$	$h_1 \cap h_2$	$\emptyset$	$h_1 h_2$	$h_1 \cup h_2$
$B_5$	$h_1$	$B_5$	$h_3$	$\emptyset$	$h_1 \cup h_3$	$h_1 \cup h_3$	$h_1 \cup h_3$
$B_6$	$\emptyset$	$B_7$	$h_3$	$h_2$	$h_2 \cup h_3$	$h_2 \cup h_3$	$h_2 \cup h_3$
$B_7$	$h_1$	$h_2$	$h_3$	$h_1 \cup h_2$	$h_1 \cup h_3$	$h_2 \cup h_3$	$\Omega$

# Calculating Products & Sums

## Illustration – Step 5

### Calculating Products

- $\{h_1\} \Rightarrow m(\mathbf{Z}_1) = m(\mathbf{A}_1) \cdot m(\mathbf{B}_1) = 0.04$
- $\{h_1\} \Rightarrow m(\mathbf{Z}_2) = m(\mathbf{A}_1) \cdot m(\mathbf{B}_5) = 0.08$
- ...
- $\{h_1 \cup h_2 \cup h_3\} \Rightarrow m(\mathbf{Z}_{11}) = m(\mathbf{A}_7) \cdot m(\mathbf{B}_7) = 0.02$

### Calculating Sum(s)

- Just  $\{h_1\} \Rightarrow \sum_{k=1}^6 m(\mathbf{Z}_k) = 0.54$

$\cap$	$A_1$	$A_2$	$A_4$	$A_7$
$B_1$	$h_1$	$\emptyset$	$h_1$	$h_1$
$B_3$	$\emptyset$	$\emptyset$	$\emptyset$	$h_3$
$B_5$	$h_1$	$\emptyset$	$h_1$	$h_1 \cup h_3$
$B_7$	$h_1$	$h_2$	$h_1 \cup h_2$	$\Omega$

$\bullet$	$A_1$	$A_2$	$A_4$	$A_7$
$B_1$	0.04	x	0.12	0.02
$B_3$	x	x	x	0.02
$B_5$	0.08	x	0.24	0.04
$B_7$	0.04	0.02	0.12	0.02

# Combining Basic Assignments

## Illustration – Step ⑥

### Sum of Product

- Example: hypothesis  $h_1$  again

$$\rightarrow \sum_{k=1}^6 m(\mathbf{Z}_k) = 0.54$$

### Calculating the Focal Sum

- Sum of all basic assignment products

$$\rightarrow \sum_{k=1}^{11} m(\mathbf{Z}_k) = 0.76$$

### Basic Assignment of the Comb. Hypothesis

- Example: hypothesis  $h_1$

$$\rightarrow m(\{h_1\}) = \frac{\sum_{k=1}^6 m(\mathbf{Z}_k)}{\sum_{k=1}^{11} m(\mathbf{Z}_k)} \approx 0.7105$$

# Measures of Combined Hypotheses

## Illustration – Step 7

### Evidence Measures

- Input by Step 6 →  $m(\mathbf{Z}_k)$
- Output by calculus of Step 2 →  $bel(\mathbf{Z}_k), pl(\mathbf{Z}_k)$
- Ranking according to  $pl(\mathbf{Z}_k)$  & certainty

$2^\Omega$	$m$	$bel$	$pl$
$\Omega$	<b>0.0263</b>	1	1
$\{h_1 \cup h_2\}$	<b>0.1579</b>	0.8947	0.9737
$\{h_1 \cup h_3\}$	<b>0.0526</b>	0.7895	0.9737
$\{h_1\}$	<b>0.7105</b>	0.7105	0.9471
$\{h_2\}$	<b>0.0263</b>	0.0263	0.2105
$\{h_3\}$	<b>0.0263</b>	0.0263	0.1053

# Interpretation

## Illustration – Step 8

### Interpretation

- Which fault may be responsible for the serious changes of system properties?
- Probabilistic approach → blames  $h_1$  alone
- Dempster-Shafer approach → points  $h_1$  and gives a hint to  $h_2$
- Different mappings  $\Omega \rightarrow [0, 1]$  versus  $2^\Omega \rightarrow [0, 1]$

$2^\Omega$	$m$	$bel$	$pl$
$\Omega$	<b>0.0263</b>	1	1
$\{h_1 \cup h_2\}$	<b>0.1579</b>	0.8947	0.9737
$\{h_1 \cup h_3\}$	<b>0.0526</b>	0.7895	0.9737
$\{h_1\}$	<b>0.7105</b>	0.7105	0.9471
$\{h_2\}$	<b>0.0263</b>	0.0263	0.2105
$\{h_3\}$	<b>0.0263</b>	0.0263	0.1053

# Outline

## Part 3

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### Part 1 – Fundamentals

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# Fault Tree Analysis

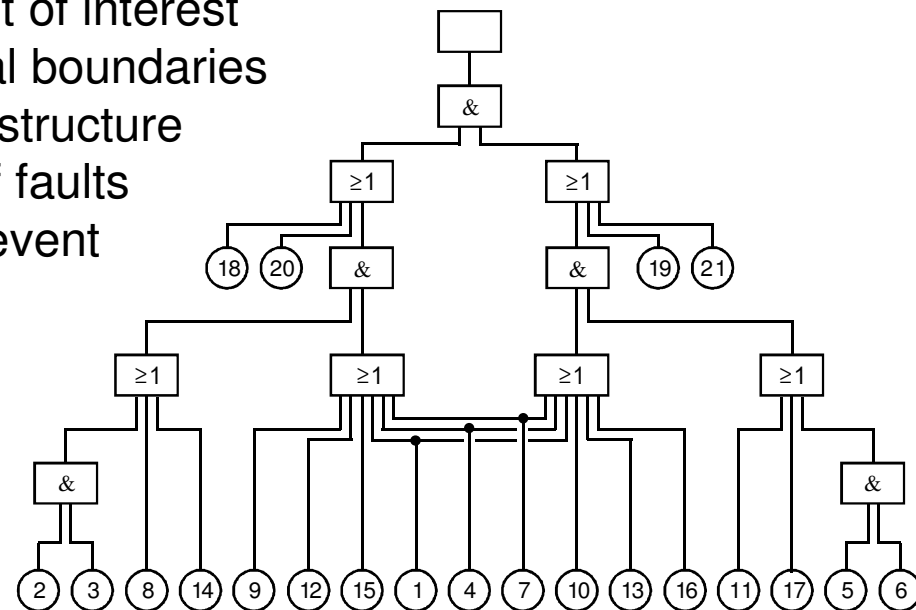
## Brief Introduction

### Detailed Introduction

- IEC 61025
- Proceedings  → references

### Four Steps of the FTA

- Step ① – Define the top event of interest
- Step ② – Define the analytical boundaries
- Step ③ – Define the tree-top structure
- Step ④ – Develop the path of faults for each branch to the basic event





# Dempster-Shafer FTA Approach

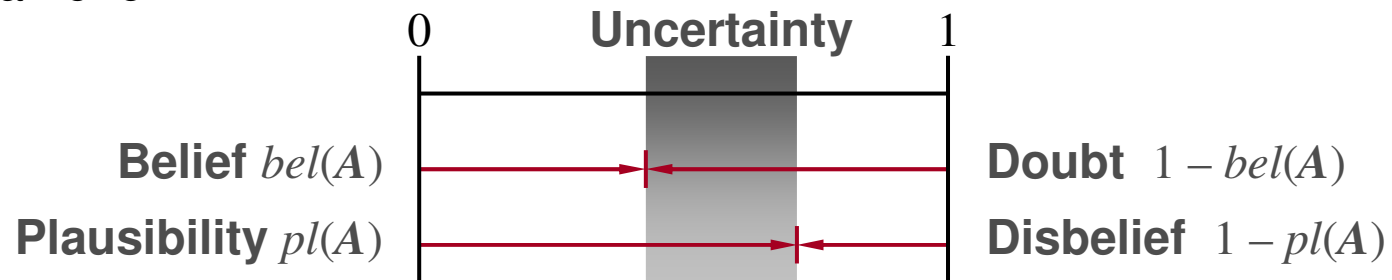
## Fault Tree Analysis

### Scenario

- $\Omega = \{h_1, h_2, h_3\} \rightarrow \{\text{"state occurs"}, \text{"uncertain"}, \text{"state does not occur"}\}$
- Gates  $\rightarrow$  **And** or **Or**
- Inputs  $\rightarrow$  e.g. two states,  $m(A)$  and  $m(B)$
- Output  $\rightarrow$  state  $m(Z)$

### The Guth Approach to DS-FTA

- $m(A_1) \equiv bel(A)$
- $m(A_2) \equiv pl(A) - bel(A)$
- $m(A_3) \equiv 1 - pl(A)$
- $m(A_1) + m(A_2) + m(A_3) = 1$
- Same for  $B$



# Dempster-Shafer FTA Approach

## Fault Tree Analysis

### AND and OR Combination

- Similar to min/max operations

### AND Gate According to Step 5

- $m(\mathbf{Z}_1) = m(\mathbf{A}_1) m(\mathbf{B}_1)$
- $m(\mathbf{Z}_2) = m(\mathbf{A}_1) m(\mathbf{B}_2) + m(\mathbf{A}_2) m(\mathbf{B}_1) + m(\mathbf{A}_2) m(\mathbf{B}_2)$
- $m(\mathbf{Z}_3) = \dots = m(\mathbf{A}_1) m(\mathbf{B}_3) + m(\mathbf{A}_2) m(\mathbf{B}_3) + m(\mathbf{A}_3)$

### OR Gate According to Step 5

- $m(\mathbf{Z}_1) = \dots = m(\mathbf{A}_1) + m(\mathbf{A}_2) m(\mathbf{B}_1) + m(\mathbf{A}_3) m(\mathbf{B}_1)$
- $m(\mathbf{Z}_2) = m(\mathbf{A}_2) m(\mathbf{B}_2) + m(\mathbf{A}_2) m(\mathbf{B}_3) + m(\mathbf{A}_3) m(\mathbf{B}_2)$
- $m(\mathbf{Z}_3) = m(\mathbf{A}_3) m(\mathbf{B}_3)$

And	$A_1$	$A_2$	$A_3$
$B_1$	$h_1$	$h_2$	$h_3$
$B_2$	$h_2$	$h_2$	$h_3$
$B_3$	$h_3$	$h_3$	$h_3$

Or	$A_1$	$A_2$	$A_3$
$B_1$	$h_1$	$h_1$	$h_1$
$B_2$	$h_1$	$h_2$	$h_2$
$B_3$	$h_1$	$h_2$	$h_3$

# Dempster-Shafer FTA Approach

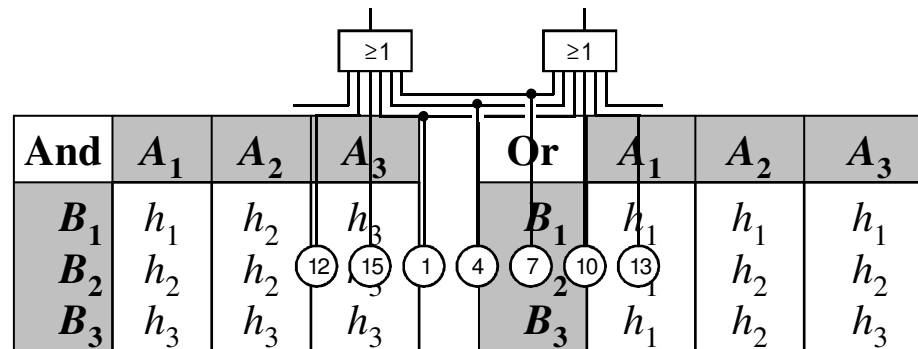
## Fault Tree Analysis

### Develop the path

- Output of the lower gate  $\rightarrow m(Z_1), m(Z_2), m(Z_3)$
- Input of the next upper gate  $\rightarrow m(A_1), m(A_2), m(A_3)$

### Criticism

- Interval arithmetic is more concise and efficient in operation than DST  
 $\rightarrow$  However, fault tree structure may cause trouble with the **sub-distributivity property of subtraction operations** as known from the fuzzy FTA
- Multistate modelling, upper/lower probs, Bayesian networks ...



# Event Tree Analysis

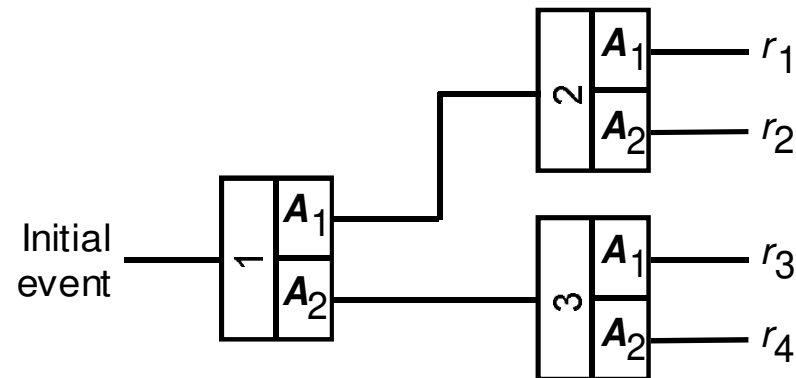
## Brief Introduction

### Detailed Introduction

- IEC 62502
- Proceedings  → references

### Five Steps of the ETA

- Step ① – List all possible initiating events
- Step ② – Identify functional responses
- Step ③ – Define failure sequences
- Step ④ – Assign **probabilities** to each step
- Step ⑤ – Calculate the total probability of occurrence for each sequence



# Dempster-Shafer ETA Approach

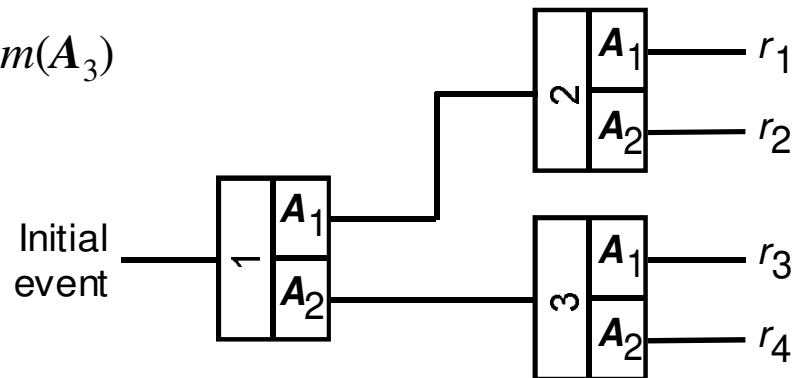
## Event Tree Analysis

### Scenario

- $\Omega = \{h_1, h_2, h_3\} \rightarrow \{\text{"failure"}, \text{"no failure"}, \text{"uncertain"}\}$
- Inputs  $\rightarrow$  data source gives 3 values  $m(A_1), m(A_2), m(A_3)$
- Output 1  $\rightarrow$  evidence measures for "failure"
- Output 2  $\rightarrow$  evidence measures for "no failure"

### Evidence Measures

- $bel(Z_1) = m(A_1)$
- $pl(Z_1) = m(A_1) + m(A_3)$
- $bel(Z_2) = m(A_2)$
- $pl(Z_2) = m(A_2) + m(A_3)$

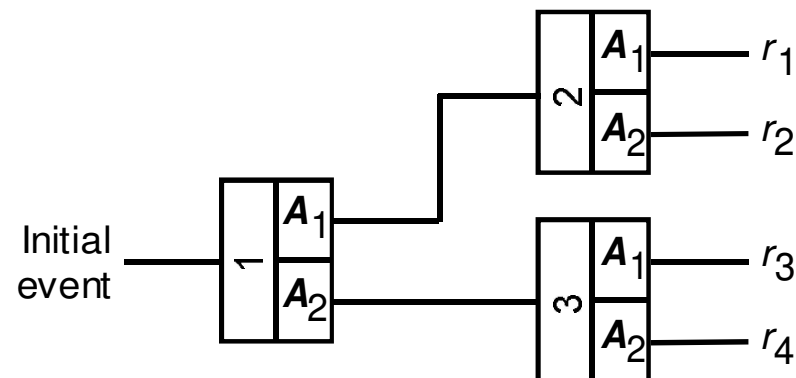


# Dempster-Shafer ETA Approach

## Event Tree Analysis

### Procedure

- Calculating evidence measures of every bifurcation of the ET
- Then applying **interval arithmetic**
- More details → RCM




# Reliability Centred Maintenance

## Brief Introduction

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### Detailed Introduction

- IEC 60300-3-11
- Proceedings  → references

### Seven Steps of the RCM Process

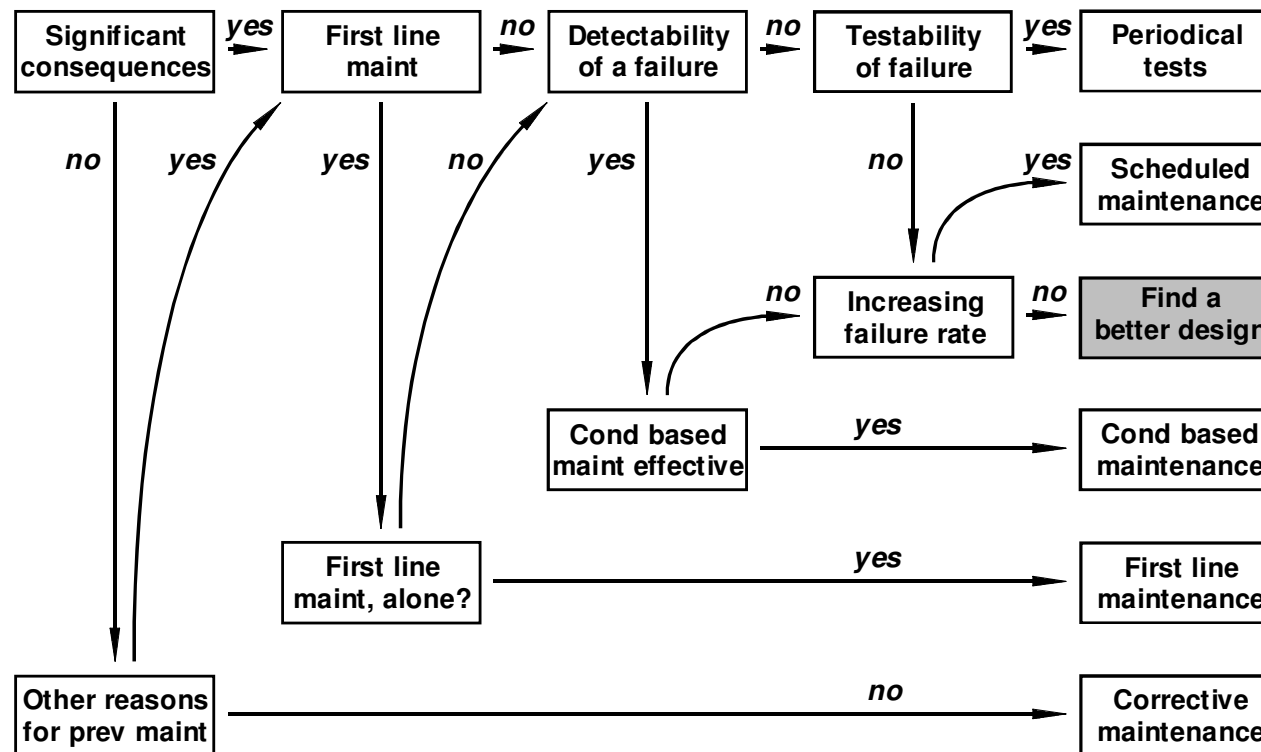
- Step ① – Establishing an expert group
- Step ② – Functional breakdown of the system
- Step ③ – Conducting FMECA
- Step ④ – Collecting of data
- Step ⑤ – Tailoring the RCM decision diagram
- Step ⑥ – Applying the RCM decision diagram
- Step ⑦ – Documenting results

# Brief Introduction

## Reliability-centred Maintenance

### RCM Decision Diagram – Objective

- Find a suitable strategy → component, module, system
- Framework of **eight** questions, **six** strategies



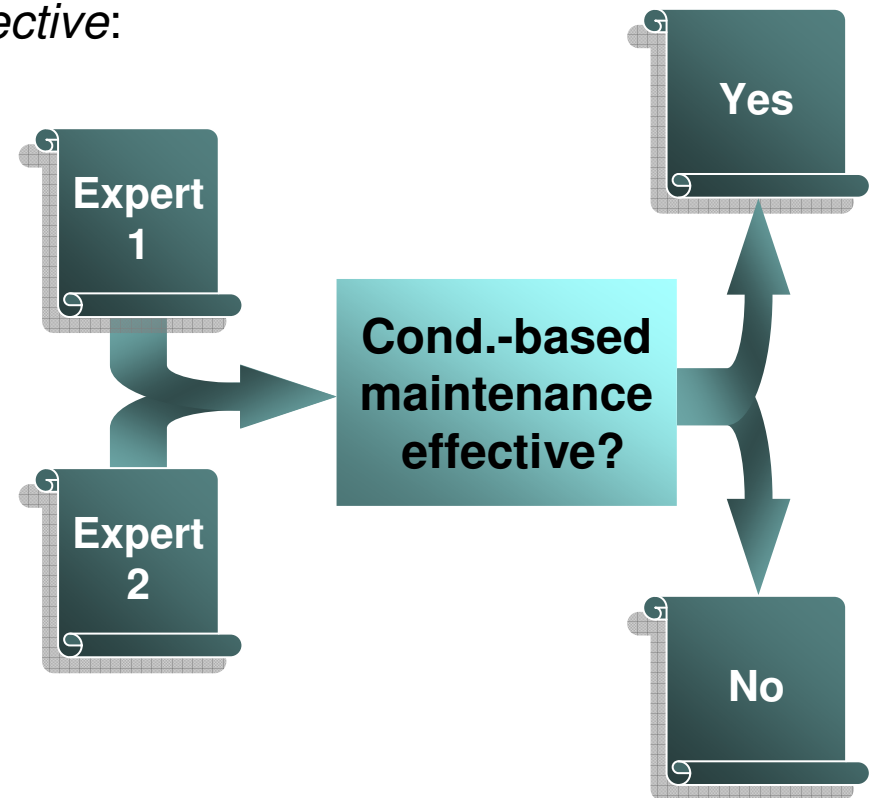


# Expert Assessment

## Reliability-centred Maintenance

### DS-RCM Example

- *Condition-based maintenance effective:*  
Do methods exist for effective condition monitoring so that an item failure can be avoided?
- Two answers
- Two experts (example)  
→ two statements

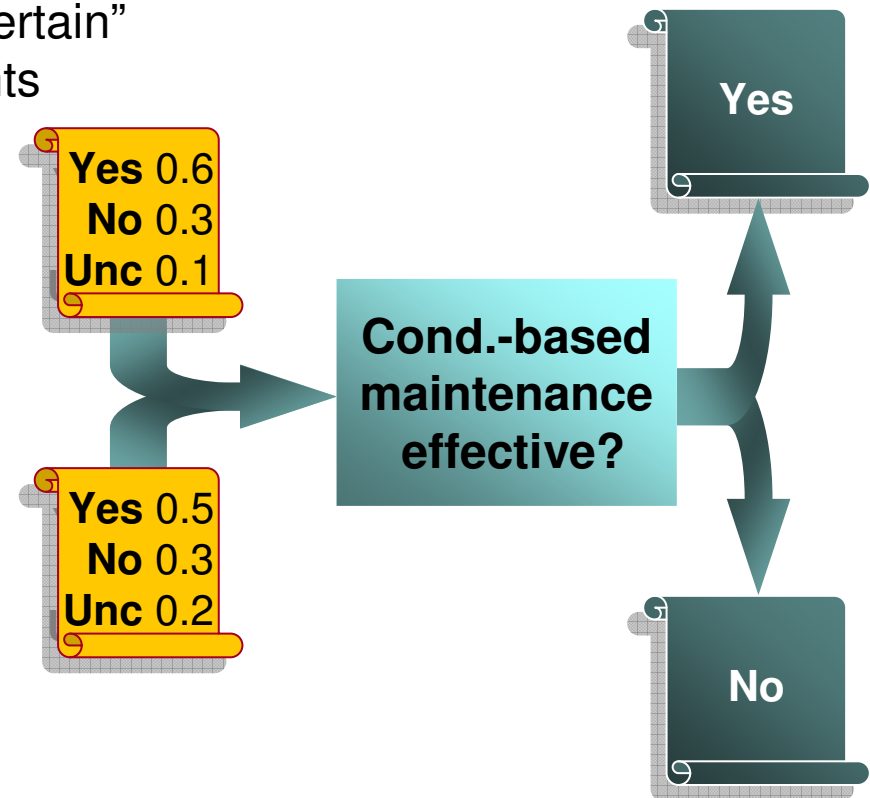


# Input & Output

## Reliability-centred Maintenance

### Input

- Statements → “yes”, “no”, or “uncertain”
- Quantification → basic assignments



# Input & Output

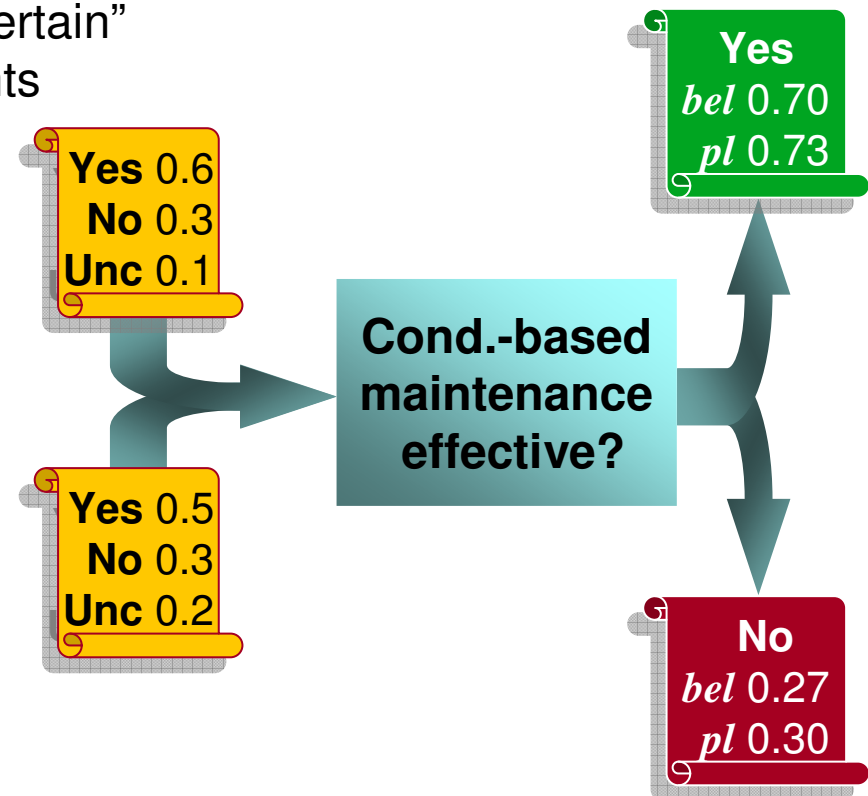
## Reliability-centred Maintenance

### Input

- Statements → “yes”, “no”, or “uncertain”
- Quantification → basic assignments

### Output

- Values of evidential functions
- Certainty
  - 70% in “yes”
  - 27% in “no”
- Uncertainty
  - 3%



# Weighted Recommendations

## Reliability-centred Maintenance

### Input

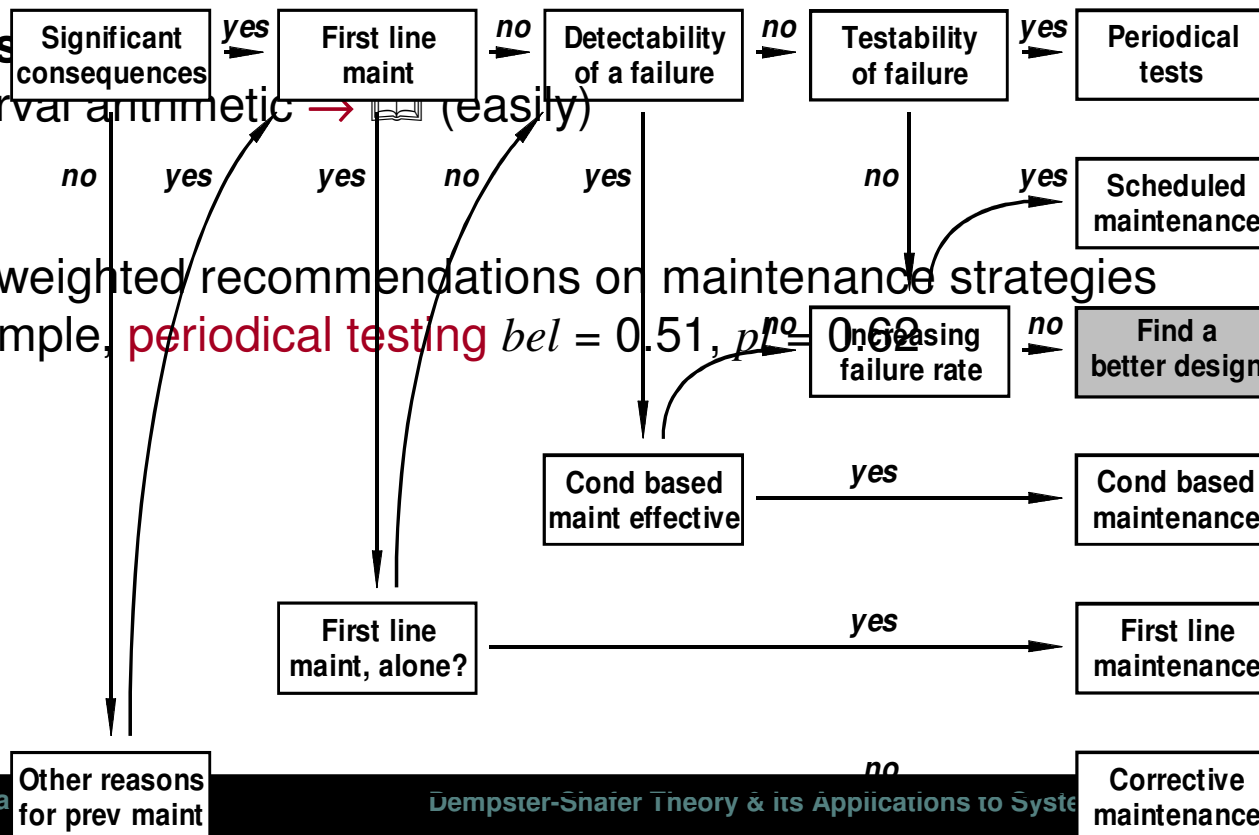
- Eight results of every “yes” or “no” decision  
→ values of evidential functions  $bel$  and  $pl$

### Calculus

- Interval arithmetic → (easily)

### Output

- Six weighted recommendations on maintenance strategies
- Example, **periodical testing**  $bel = 0.51$ ,  $pl = 0.69$




# Further Analyses

## Some Hints

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### Failure Mode, Effects and Criticality Analysis

- IEC 60812
- Dempster-Shafer approach → Section 4.1 Proceedings 

### Preliminary/Potential Hazard Analysis

- ... same holds for PHA

# Outline

## Part 4

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### Part 1 – Fundamentals

- History
- Scenario
- Interpretations

### Part 2 – Illustration

- The DS calculus in eight steps

### Part 3 – Applications to System Safety & Reliability Modelling

- FTA – Fault Tree Analysis
- ETA – Event Tree Analysis
- RCM – Reliability-centred Maintenance
- Further Analyses

### Part 4 – Outroduction

- Pros & Cons

# Some Comments

## Outroduction

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### Disclaimer

- Nobody is forced to apply DST instead of Probability Theory
- No uncertainties → no DS modelling recommended (?)
- Prefer modelling uncertainties by probabilities? → apply probabilities
- Prefer ... interval arithmetic? → apply interval arithmetic
- Prefer ... fuzzy sets? → apply fuzzy sets
- Applying DST is an **option**, not an obligation

# Disadvantages of the DST

## Outroduction

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### Disadvantages (also valid for Probabilities)

- *Lack of introspection or assessment strategies*  
unreasonable requirement for precision →  $m$   
difficult to determine with necessary precision
- *Instability*  
estimated  $m$  may be influenced by the conditions of its estimation
- *Ambiguity*  
ambiguous or imprecise judgement could not be expressed  
by the evidence measures

### Disadvantages

- Frame of discernment  $\Omega$  → given  $k$  hypotheses → up to  $2^k$  elements  
larger number of values → than after the Probability Theory
- DST does not offer a procedure for implementation of a diagnostic system



# Advantages of the DST

## Outroduction

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### Advantages

- Calculus describes & quantifies the **subjective** viewpoint as an assessment for an unknown **objective** fact
- Applying DST is an **option**, not an obligation
- “If the only tool you have is a hammer, you tend to see every problem as a nail.” 📖 Abraham Maslow



# **Fundamentals of the Dempster-Shafer Theory and its Applications to System Safety and Reliability Modelling**

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